PARETO OPTIMAL SOLUTIONS TO MULTI OBJECTIVE LINEAR PROGRAMMING PROBLEM WITH FUZZY GOALS USING TRADE OFF RATIOS

A. Garai*[1], P. Mandal[2], Dr. T. K. Roy[3]

[1]Department of Mathematics, Sonarpur Mahavidyalaya, S. B. Sarani, Rajpur, Kolkata-700149

[2,3]Department of Mathematics, IIEST, Shibpur, Howrah-711103
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ABSTRACT

In this paper, we propose a technique to find Pareto optimal solution to multi objective linear programming problem with fuzzy goals using trade off ratios among membership functions of objectives.

Keywords: *multi objective optimisation, fuzzy multi objective optimisation, fuzzy goals, interactive method, Pareto optimal solutions, trade off ratios.*

1. INTRODUCTION

We consider the following multi objective linear programming problem (MOLPP) with k objective functions $z_i(x)$, i = 1. . . K as

$$Min\ \mathrm{z(x)} = (\mathrm{z}_1(x), \mathrm{z}_2(x), ..., \mathrm{z}_k(x))^T \quad (1)$$
$$subject\ \text{to } \mathrm{x} \in \mathrm{X}$$

Here $X = \{x \in R^n : Ax \le b, x \ge 0\}, b = (b_1, b_2, ..., b_m) \in R^m\}$ and A is $m \times n$ matrix. '*R*' denotes the set of real numbers. By assuming that the Decision maker (DM) [4] has imprecise aspiration level for each objective function $z_i(x)$, i = 1. . . K of model (1), several methods have been proposed in literature to characterize Pareto optimal solutions to MOLPP (1).

One such approach involves usage of fuzzy set theory introduced by Zadeh (1965) [6]. As suggested by Zimmermann [2] and his successors, here MOLPP is converted into single objective optimization problem to attain M-Pareto optimal solution and then Pareto optimal solution by using the fuzziness of the DM's aspiration with respect to the goals of imprecise objective functions. Recently the concepts of evolutionary techniques such as particle swarm optimisation and genetic algorithm were proposed to solve multi objective optimisation problems. Suitable definition of continuity and differentiability of fuzzy-valued function have also been used to study multi objective programming problems with fuzzy-valued objective functions. More solution concepts and techniques such as the KKT optimality conditions, the necessary and sufficient conditions of Pareto-optimality, and approach by Tanaka et al (1984) [5] for imprecise multi objective programming problems have been discussed in literature.

Again in MOLPP in fuzzy environment as suggested by Zimmermann [4] and his successors, it has been implicitly assumed that the fuzzy decision by Bellman and Zadeh (1970) [11] is the proper representation of the fuzzy preferences by DM. But these approaches are preferable only when DM feels that fuzzy decision is appropriate for combining fuzzy goals and/or constraints [3]. The occurrence of such situation is rarely in practice.

Sakawa et al (1987) [7] suggested an interactive method to solve fuzzy MOLPP. In their method, Sakawa et al (1987) [7] identified that through the interaction with the decision maker (DM), the fuzzy goals of the DM for each of the objective functions are quantified by eliciting corresponding membership functions. After determining the membership functions, in order to generate a candidate for the M-Pareto optimal solution and finally Pareto optimal solution, the DM specifies reference membership value for each imprecise objective function. In their method, Sakawa et al (1987) [8] suggested all initial reference membership levels at unity. Then M-Pareto optimal solution is obtained by minimizing the distance between membership functions and corresponding reference membership levels for all objectives.

'Cut your coat according to your cloth.' In real life, all of us do not reach the same top together. And unrealistic aspiration levels increase the pressure unnecessarily and may lead to total collapse. So reference membership level for objectives should be realistic and achievable.

Corresponding Author: A. Garai[1]*

We arrange the paper as follows: definitions are given; next an algorithm is proposed to find M-Pareto optimal solution and finally Pareto optimal solution to MOLPP with fuzzy goals; then one numerical example illustrates proposed algorithm; we compare this solution with results obtained by using max min operator as suggested by Bellman and Zadeh [11], then extended by Zimmermann [2] and Sakawa's method [9]. Finally we draw conclusions.

2 DEFINITIONS

We know that trade studies are essentially decision-making exercises. A trade off is a situation that involves losing one aspect of an objective in return for gaining quality or aspect of another objective. More colloquially, if one thing increases, some other thing must decrease.

Definition 2.1 (Sakawa [10]): A decision plan $x_0 \in X$ is said to be an M-Pareto optimal solution to the MOLPP model (1) if there does not exist another $y \in X$ such that $\mu_i(z_i(x_0)) \le \mu_i(z_i(y)) \forall i, i \ne j$ and $\mu_j(z_j(x_0)) < \mu_j(z_j(y))$ for at least one j.

Definition 2.2 (Sakawa [10]): A decision plan $x_0 \in X$ is said to be a Pareto-optimal solution to the MOLPP model (1) if there does not exist another $y \in X$ such that $z_i(y) \le z_i(x_0)$ for all i, i ≠ j and $z_j(y) < z_j(x_0)$ for at least one j.

3 ALGORITHM TO FIND PARETO OPTIMAL SOLUTION OF MOLPP WITH FUZZY GOALS

Within the scope of the multi objective optimisation, Pareto-optimality of a solution is necessary condition in order to guarantee rationality of a decision [12]. And utilization of DM's preference is highly recommended in determining membership function of fuzzy objective. In minimization problem, one fuzzy goal of DM may be to achieve "substantially less" than some value £. This type of statement can be quantified by eliciting membership function and corresponding reference membership value.

We can integrate above ideas into a general framework and develop an algorithm to obtain Pareto optimal solution to MOLPP with fuzzy goals.

The steps of the proposed algorithm are as follows:

Step 1 Calculate individual maximum and minimum of each objective function under the given constraints. If unbounded solutions are encountered to some objective functions, very large numbers (comparing with the rest) may be taken as extreme values.

Step 2 Then we ask DM to specify goals and tolerances for each of the fuzzy objectives so that membership functions $\mu_i(z_i(x)), i = 1 \ldots k$ of each objective function $z_i(x), i = 1 \ldots k$ can be constructed. If DM is not available or he/she does not specify goal and/or tolerance of objective(s), we may construct membership function(s) by using the individual minimum and maximum value(s) of that objective(s) or choose suitable goal(s) and/or tolerance(s).

Step 3 Next we choose one objective function, say $z_t, t = 1 \ldots k$ arbitrarily. By using chain rule, we compute the trade off ratios between corresponding membership function $\mu_t(z_t(x))$ of $z_t(x)$ and other membership functions $\mu_j(z_j(x)), j = 1 \ldots k, j \ne t$ of $z_j(x), j = 1 \ldots k, j \ne t$ respectively as follows:

$$-\frac{\partial \mu_t(z_t(x))}{\partial \mu_j(z_j(x))} = -\frac{\partial \mu_t(z_t(x))}{\partial z_t(x)} \frac{\partial z_t(x)}{\partial z_j(x)} \left(\frac{\partial \mu_j(z_j(x))}{\partial z_j(x)} \right)^{-1}, \quad j = 1 \ldots k, j \ne t.$$

Step 4 Then we determine reference membership level for each objective function as follows:
First derive a set of numbers, say $\bar{\mu}_1, \bar{\mu}_2 \ldots \bar{\mu}_k$, using the formula

$$\bar{\mu}_j = \left| \left(-\frac{\partial \mu_t(z_t(x))}{\partial \mu_j(z_j(x))} \right)^{-1} \right| \bar{\mu}_t, \; j = 1 \ldots k, j \ne t,$$

Here take $\bar{\mu}_t = 1$. Set $\hat{\mu} = \max\{\bar{\mu}_1, \bar{\mu}_2 \ldots \bar{\mu}_k\}$. Clearly $\hat{\mu} \ne 0$. Then reference membership level $\hat{\mu}_i$ of ith objective function $z_i(x)$ may be determined by the formula $\hat{\mu}_i = \bar{\mu}_i / \hat{\mu}, \forall i = 1 \ldots k, (\because \hat{\mu} \ne 0)$. In this way we use trade off ratios to compute reference membership levels in place of arbitrary choice.

Step 5 Using these reference membership levels, we now determine M-Pareto optimal solution by solving the corresponding mini-max problem as follows

$$\min v$$
$$\text{subject to} \quad \hat{\mu}_i - \mu_i(z_i(x)) \le v, \forall i = 1 \ldots k, v \ge 0, x \in X.$$

Step 6 Let x^* be the M-Pareto optimum solution to the problem at step 5. To test the Pareto optimality of the current solution x^*, solve the following problem

$$\max \sum_{i=1}^{k} \varepsilon_i$$

$$\textit{subject to } \mu_i(z_i(x)) - \varepsilon_i \geq \mu_i(z_i(x^*)), \forall i = 1 \ldots k, x \in X.$$

Let $\bar{x}$ and $\bar{\varepsilon}$ are optimal solutions to this problem. Then two cases may arise:

a) If all $\bar{\varepsilon}_i = 0$, then x^* is Pareto optimal solution of the MOLPP with fuzzy goals.

b) If at least one $\bar{\varepsilon}_i > 0$, then x^* is not Pareto optimal solution of the MOLPP with fuzzy goals. Instead of x^*, $\bar{x}$ is Pareto optimal solution to the MOLPP with fuzzy goals. This completes the algorithm.

4. NUMERICAL EXAMPLES

Based on the algorithm discussed above, we now consider one MOLPP with fuzzy goals as follows

$$\textit{fuzzy max} \quad z_1 = 5x_1 + 5x_2$$

$$\textit{fuzzy min} \quad z_2 = 5x_1 + x_2$$

$$\textit{fuzzy } \max \quad z_3 = 3x_1 - 8x_2$$

$$\textit{subject to}$$

$$5x_1 + 7x_2 \leq 12, 9x_1 + x_2 \leq 10, -5x_1 + 3x_2 \leq 3, x_1, x_2 \geq 0.$$

Solution technique 4.1: We first solve the problem according to our proposed algorithm. The steps are as follows

Step 1 First we compute individual maximum and minimum of each objective function under the given constraints.

Table 1 Individual maximum and minimum

Objective functions	Individual maximum	Individual Minimum
z_1	10	0
z_2	6	0
z_3	3.333	-11.1

Step 2 Suppose that the DM specifies neither goal nor tolerance of any objective function. So we construct membership functions of fuzzy objectives by using those individual maximum and minimum values as follows

$$\mu_1(z_1(x)) = \begin{cases} 0, \text{ if } z_1(x) \leq 0 \\ \dfrac{z_1 - 0}{10}, \text{ if } 0 \leq z_1(x) \leq 10, \\ 1, \text{ if } z_1(x) \geq 10 \end{cases} \quad \mu_2(z_2(x)) = \begin{cases} 0, \text{ if } z_2(x) \geq 6 \\ \dfrac{6 - z_2}{6}, \text{ if } 0 \leq z_2(x) \leq 6, \\ 1, \text{ if } z_2(x) \leq 0 \end{cases} \quad \mu_3(z_3(x)) = \begin{cases} 0, \text{ if } z_3(x) \leq -11.1 \\ \dfrac{z_3 + 11.1}{14.43}, \text{ if } -11.1 \leq z_3(x) \leq 3.33 \\ 1, \text{ if } z_3(x) \geq 3.33 \end{cases}$$

,

Step 3 Next we arbitrarily select one objective function, suppose $z_1(x)$. Using chain rule, we compute trade off ratios between membership function $\mu_1(z_1(x))$ of objective function $z_1(x)$ and membership functions $\mu_2(z_2(x))$ and $\mu_3(z_3(x))$ of objective functions $z_2(x)$ and $z_3(x)$ respectively as

$$-\frac{\partial \mu_1(z_1(x))}{\partial \mu_2(z_2(x))} = -\frac{\partial \mu_1(z_1(x))}{\partial (z_1(x)} \frac{\partial z_1(x)}{\partial z_2(x)} \left(\frac{\partial \mu_2(z_2(x))}{\partial z_2(x)} \right)^{-1} = 0.767 \text{ and}$$

$$-\frac{\partial \mu_1(z_1(x))}{\partial \mu_3(z_3(x))} = -\frac{\partial \mu_1(z_1(x))}{\partial z_1(x)} \frac{\partial z_1(x)}{\partial z_3(x)} \left(\frac{\partial \mu_3(z_3(x))}{\partial z_3(x)} \right)^{-1} = 0.671.$$

Step 4 Set $\bar{\mu}_1 = 1$. Then we obtain $\bar{\mu}_2$ *and* $\bar{\mu}_3$ as $\bar{\mu}_2 = 1.304$ *and* $\bar{\mu}_3 = 1.490$. Therefore $\hat{\mu} = 1.490$. Hence (using the formula $\hat{\mu}_i = {}^{\bar{\mu}_i}\!/_{\hat{\mu}}$, $\forall i = 1, 2, 3$) reference membership levels for objective functions $z_1(x), z_2(x), z_3(x)$ are $\hat{\mu}_1 = 0.671, \hat{\mu}_2 = 0.875, \hat{\mu}_3 = 1$.

Step 5 For these reference membership levels, we construct corresponding mini-max problem as

$$\min v$$
$$subject\ to$$
$$0.671-\left(\frac{5x_1+5x_2-0}{10}\right)\le v, 0.875-\left(\frac{6-(5x_1+x_2)}{6}\right)\le v,$$
$$1-\left(\frac{3x_1-8x_2+11.1}{14.43}\right)\le v, 5x_1+7x_2\le 12, 9x_1+x_2\le 10,$$
$$-5x_1+3x_2\le 3, x_1, x_2, v\ge 0.$$

Step 6 Using Lingo 15.0.32, we obtain M-Pareto optimal solution to the above problem as

$$v^*=0.299,\ x_1^*=0.451,\ x_2^*=0.293, z_1(x^*)=3.72,\ z_2(x^*)=2.548,\ z_3(x^*)=-0.991,$$
$$\mu_1(z_1(x^*))=0.372,\ \mu_2(z_2(x^*))=0.575,\ \mu_3(z_3(x^*))=0.700.$$

Now to test the Pareto optimality of this M-Pareto optimal solution, we solve the following problem

$$\max imize\quad \varepsilon_1+\varepsilon_2+\varepsilon_3$$
$$subject\ to$$
$$\left(\frac{5x_1+5x_2-0}{10}\right)-\varepsilon_1\ge 0.372, \left(\frac{6-(5x_1+x_2)}{6}\right)-\varepsilon_2\ge 0.575,$$
$$\left(\frac{3x_1-8x_2+11.1}{14.43}\right)-\varepsilon_3\ge 0.700, 5x_1+7x_2\le 12, 9x_1+x_2\le 10,$$
$$-5x_1+3x_2\le 3,\ x_1, x_2, \varepsilon_1, \varepsilon_2, \varepsilon_3\ge 0.$$

Using Lingo 15.0.32, we obtain the Pareto optimal solutions $\overline{x}$ and $\overline{\varepsilon}$ as

$$\overline{\varepsilon}_1=\overline{\varepsilon}_2=\overline{\varepsilon}_3=0, \overline{x}_1=0.451,\ \overline{x}_2=0.293,\ z_1(\overline{x})=3.72,\ z_2(\overline{x})=2.548,\ z_3(\overline{x})=-0.991,$$
$$\mu_1(z_1(\overline{x}))=0.372,\ \mu_2(z_2(\overline{x}))=0.575,\ \mu_3(z_3(\overline{x}))=0.700.$$

Here $\overline{\varepsilon}_i=0\ \forall$ i = 1, 2, 3 and so the solution x^* obtained from step 5 is Pareto optimal solution to the MOLPP with fuzzy goals.

Solution technique 4.2: We now solve the same problem using max min operator as suggested by Bellman and Zadeh [11] and later on extended by Zimmermann [1]. Using max min operator, the MOLPP with fuzzy goals is converted into single objective linear programming problem as follows

$$\max \lambda$$
$$subject\ to$$
$$\frac{5x_1+5x_2-0}{10}\ge\lambda,\ \frac{6-(5x_1+x_2)}{6}\ge\lambda,$$
$$\frac{3x_1-8x_2+11.1}{14.43}\ge\lambda, 5x_1+7x_2\le 12,\ 9x_1+x_2\le 10,$$
$$-5x_1+3x_2\le 3,\ 0\le\lambda\le 1,\ x_1, x_2\ge 0.$$

Using Lingo 15.0.32, we obtain the M-Pareto optimal solution x^* as

$$\lambda^*=0.526,\ x_1^*=0.447,\ x_2^*=0.606, z_1(x^*)=5.265,\ z_2(x^*)=2.841,\ z_3(x^*)=-3.507,$$
$$\mu_1(z_1(x^*))=0.526,\ \mu_2(z_2(x^*))=0.526,\ \mu_3(z_3(x^*))=0.526.$$

Now to test Pareto optimality of this M-Pareto optimal solution, we solve the following problem

$$\max\quad \sum_{i=1}^{3}\varepsilon_i$$
$$subject\ to$$
$$\frac{(5x_1+5x_2)-0}{10}-\varepsilon_1\ge 0.526, \frac{6-(5x_1+x_2)}{6}-\varepsilon_2\ge 0.526,$$
$$\frac{(3x_1-8x_2)+11.1}{14.43}-\varepsilon_3\ge 0.526, 5x_1+7x_2\le 12,\quad 9x_1+x_2\le 10,$$
$$-5x_1+3x_2\le 3, x_1, x_2, \varepsilon_1, \varepsilon_2, \varepsilon_3\ge 0.$$

Using Lingo 15.0.32, we obtain optimal solutions $\overline{x}$ and $\overline{\varepsilon}$ as follows

$$\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = \bar{\varepsilon}_3 = 0,\ \bar{x}_1 = 0.447,\ \bar{x}_2 = 0.606,\ z_1(\bar{x}) = 5.265,\ z_2(\bar{x}) = 2.841,\ z_3(\bar{x}) = -3.507,$$
$$\mu_1(z_1(\bar{x})) = 0.526,\ \mu_2(z_2(\bar{x})) = 0.526,\ \mu_3(z_3(\bar{x})) = 0.526.$$

Here $\bar{\varepsilon}_i = 0 \forall i = 1,2,3.$ Hence the M-Pareto optimal solution x^* is Pareto optimal solution to MOLPP with fuzzy goals.

Solution technique 4.3: Next we use Sakawa's method (1987) of finding Pareto optimal solution to convert MOLPP with fuzzy goals into single objective linear programming problem as follows

$$\min \quad \lambda$$

subject to

$$1-\frac{5x_1+5x_2-0}{10} \geq \lambda, 1-\frac{6-(5x_1+x_2)}{6} \geq \lambda, 1-\frac{3x_1-8x_2+11.1}{14.43} \geq \lambda, 5x_1+7x_2 \leq 12,\ 9x_1+x_2 \leq 10,\ -5x_1+3x_2 \leq 3,\ x_1, x_2 \geq 0.$$

Using Lingo 15.0.32, we obtain the M-Pareto optimal solution as

$$\lambda^* = 0.474,\ x_1^* = 0.447,\ x_2^* = 0.606,\ z_1(x^*) = 5.265,\ z_2(x^*) = 2.841,\ z_3(x^*) = -3.507,$$
$$\mu_1(z_1(x^*)) = 0.526,\ \mu_2(z_2(x^*)) = 0.526,\ \mu_3(z_3(x^*)) = 0.526.$$

To test the Pareto optimality of this M-Pareto optimal solution, we solve the following problem

$$\max \quad \sum_{i=1}^{3} \varepsilon_i$$

$$\textit{subject to}\quad \frac{(5x_1+5x_2)-0}{10} - \varepsilon_1 \geq 0.526, \frac{6-(5x_1+x_2)}{6} - \varepsilon_2 \geq 0.526,$$
$$\frac{(3x_1-8x_2)+11.1}{14.43} - \varepsilon_3 \geq 0.526, 5x_1+7x_2 \leq 12,\quad 9x_1+x_2 \leq 10,$$
$$-5x_1+3x_2 \leq 3,\ x_1,\ x_2,\ \varepsilon_1,\ \varepsilon_2,\ \varepsilon_3 \geq 0.$$

Using Lingo 15.0.32, we obtain Pareto optimal solutions $\bar{x}$ and $\bar{\varepsilon}$ as follows

$$\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = \bar{\varepsilon}_3 = 0,\ \bar{x}_1 = 0.447,\ \bar{x}_2 = 0.606,\ z_1(\bar{x}) = 5.265,\ z_2(\bar{x}) = 2.841,\ z_3(\bar{x}) = -3.507,$$
$$\mu_1(z_1(\bar{x})) = 0.526,\ \mu_2(z_2(\bar{x})) = 0.526,\ \mu_3(z_3(\bar{x})) = 0.526.$$

Here $\bar{\varepsilon}_i = 0 \forall i = 1,2,3.$ Hence the M-Pareto optimal solution x^* is Pareto optimal solution to MOLPP with fuzzy goals.

Table 2 Membership values of objectives in different techniques

Pareto optimal solution techniques	Optimum membership values of objective functions			Average membership values
	$Z_1(x)$	$Z_2(x)$	$Z_3(x)$	
Proposed technique	0.372	0.575	0.700	*0.549*
Zimmermann's technique	0.526	0.526	0.526	0.526
Sakawa's technique	0.526	0.526	0.526	0.526

5. CONCLUSIONS

In this paper, we have proposed a technique to obtain Pareto optimal solutions to MOLPP with fuzzy goals. Sakawa et al (1987) have developed a method where all initial reference membership levels are unity, the maximum grade. But in reality, reference membership level of each objective function should not be set at same maximum grade. In fact arbitrarily expecting all objectives to attain the same goal of unity seems not very realistic. We know that aim or target values may not be same for all and all cannot perform together to attain same top level. Targets or reference levels should be realistic, achievable. Hence in this paper we have proposed an algorithm to characterize Pareto optimal solutions by using reference membership levels obtained from trade off ratios among membership functions of objectives instead of arbitrarily choosing unity as fixed reference membership levels. Moreover it saves the precious time of DM. And this method can be further extended in solving fuzzy multi objective nonlinear programming problems as well.

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AUTHORS' DETAILS

Arindam Garai
Department of Mathematics, Sonarpur Mahavidyalaya, S. B. Sarani, Rajpur, Kolkata, West Bengal, Pin-700149
e-mail: fuzzy_arindam@yahoo.com Mob.: +91-9932890115

Palash Mandal
Department of Mathematics, IIEST, Shibpur, Howrah, West Bengal, Pin-711103
e-mail: palashmandalmbss@gmail.com Mob.: +91-9874410213

Dr. Tapan Kumar Roy
Department of Mathematics, IIEST, Shibpur, Howrah, West Bengal, Pin-711103
e-mail: roy_t_k@yahoo.co.in Mob.: +91-9477419380